Limites

① $\lim\limits_{x\to 0}\left(\dfrac{3x-5}{x+2}\right) = \dfrac{3(0)-5}{0+2} = -5/2$

② $\lim\limits_{x\to 3}\left(\dfrac{x^3-3x^2+2x-6}{x+4}\right) = \dfrac{27-27+6-6}{7} = \dfrac{0}{7} = 0$

③ $\lim\limits_{x\to 8}\left(\dfrac{x^2-64}{x-8}\right) = \lim\limits_{x\to 8}\dfrac{(x-8)(x+8)}{(x-8)} = \lim\limits_{x\to 8}(x+8) = 16$

④ $\lim\limits_{x\to 2}\left(\dfrac{x^3-8}{x-2}\right) = \lim\limits_{x\to 2}\dfrac{(x-2)(x^2+2x+4)}{(x-2)}$

$= \lim\limits_{x\to 2}(x^2+2x+4) = 12$

⑤ $\lim\limits_{x\to 1}\left(\dfrac{x^2-1}{2x^2-x-1}\right) = \lim\limits_{x\to 1}\dfrac{(x+1)(x-1)}{(2x+1)(x-1)} = \lim\limits_{x\to 1}\left(\dfrac{x+1}{2x+1}\right) = 2/3$

⑥ $\lim\limits_{x\to -1}\left(\dfrac{x^2-1}{x^2+3x+2}\right) = \lim\limits_{x\to -1}\dfrac{(x+1)(x-1)}{(x+1)(x+2)} = \lim\limits_{x\to -1}\left(\dfrac{x-1}{x+2}\right) = -2$

⑦ $\lim\limits_{y\to -2}\left(\dfrac{y^3+4y^2+4y}{y^2-y-6}\right) = \lim\limits_{y\to -2}\dfrac{y(y+2)(y+2)}{(y-3)(y+2)} = \dfrac{-2(0)}{-5} = 0$

⑧ $\lim\limits_{x\to 3}\left(\dfrac{x^2-5x+6}{x^2-8x+15}\right) = \lim\limits_{x\to 3}\dfrac{(x-3)(x-2)}{(x-3)(x-5)} = \dfrac{1}{-2} = -1/2$

⑨ $\lim\limits_{x\to 1}\left(\dfrac{x^3-3x+2}{x^4-4x+3}\right)$ FACTORIZANDO NUMERADOR Y DENOMINADOR:

x^3+0x^2-3x+2

$$\begin{array}{c|cccc} & 1 & 0 & -3 & 2 \\ 1 & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \\ 1 & & 1 & 2 & \\ \hline & 1 & 2 & 0 & \end{array}$$

$(x-1)^2(x+2)$

$x^4+0x^3+0x^2-4x+3$

$$\begin{array}{c|ccccc} & 1 & 0 & 0 & -4 & 3 \\ 1 & & 1 & 1 & 1 & -3 \\ \hline & 1 & 1 & 1 & -3 & 0 \\ 1 & & 1 & 2 & 3 & \\ \hline & 1 & 2 & 3 & 0 & \end{array}$$

$(x-1)^2(x^2+2x+3)$

RETOMANDO:

$\lim\limits_{x\to 1}\dfrac{(x-1)^2(x+2)}{(x-1)^2(x^2+2x+3)} = \lim\limits_{x\to 1}\left(\dfrac{x+2}{x^2+2x+3}\right) = 1/2$

⑩ $\lim\limits_{x\to 2}\left(\dfrac{x^3-2x^2-4x+8}{x^4-8x^2+16}\right)$ PROCEDIENDO COMO EN EL EJERCICIO ANTERIOR:

x^3-2x^2-4x+8

$$\begin{array}{c|cccc} & 1 & -2 & -4 & 8 \\ 2 & & 2 & 0 & -8 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$(x-2)^2(x+2)$

$x^4+0x^3-8x^2+0x+16$

$$\begin{array}{c|ccccc} & 1 & 0 & -8 & 0 & 16 \\ 2 & & 2 & 4 & -8 & -16 \\ \hline & 1 & 2 & -4 & -8 & 0 \\ 2 & & 2 & 8 & 8 & \\ \hline & 1 & 4 & 4 & 0 & \end{array}$$

$(x-2)^2(x+2)^2$

RETOMANDO:

$\lim\limits_{x\to 2}\dfrac{(x-2)^2(x+2)}{(x-2)^2(x+2)^2} = \lim\limits_{x\to 2}\dfrac{1}{(x+2)} = 1/4$

(11) $$\lim_{x\to 2}\left(\frac{x^4+2x^3-x^2-8x-12}{x^4-2x^3-x^2+8x-12}\right)$$

FACTORIZANDO:

$$\begin{array}{c|ccccc} & 1 & 2 & -1 & -8 & -12 \\ 2 & & 2 & 8 & 14 & 12 \\ \hline & 1 & 4 & 7 & 6 & 0 \end{array} \qquad (x-2)(x^3+4x^2+7x+6)$$

$$\begin{array}{c|ccccc} & 1 & -2 & -1 & 8 & -12 \\ 2 & & 2 & 0 & -2 & 12 \\ \hline & 1 & 0 & -1 & 6 & 0 \end{array} \qquad (x-2)(x^3-x+6)$$

$$\lim_{x\to 2}\frac{(x-2)(x^3+4x^2+7x+6)}{(x-2)(x^3-x+6)} = \frac{44}{12} = 11/3$$

(12) $$\lim_{x\to 1}\left(\frac{x^6-1}{x^3-1}\right) = \lim_{x\to 1}\frac{(x^3-1)(x^3+1)}{(x^3-1)} = 2$$

(13) $$\lim_{x\to 1}\left(\frac{x^4-3x+2}{x^5-4x+3}\right)$$

FACTORIZANDO:

$$\begin{array}{c|ccccc} & 1 & 0 & 0 & -3 & 2 \\ 1 & & 1 & 1 & 1 & -2 \\ \hline & 1 & 1 & 1 & -2 & 0 \end{array} \qquad (x-1)(x^3+x^2+x-2)$$

$$\begin{array}{c|cccccc} & 1 & 0 & 0 & 0 & -4 & 3 \\ 1 & & 1 & 1 & 1 & 1 & -3 \\ \hline & 1 & 1 & 1 & 1 & -3 & 0 \end{array} \qquad (x-1)(x^4+x^3+x^2+x-3)$$

RETOMANDO:

$$\lim_{x\to 1}\frac{(x-1)(x^3+x^2+x-2)}{(x-1)(x^4+x^3+x^2+x-3)} = \frac{3-2}{4-3} = 1$$

(14) $$\lim_{x\to 2}\frac{(x^2-x-2)^{20}}{(x^3-12x+16)^{10}}$$

$$\begin{array}{c|cccc} & 1 & 0 & -12 & 16 \\ 2 & & 2 & 4 & -16 \\ \hline & 1 & 2 & -8 & 0 \end{array} \qquad \begin{aligned}&(x-2)(x^2+2x-8)\\ &= (x-2)(x-2)(x+4)\end{aligned}$$

$$\lim_{x\to 2}\frac{[(x-2)(x+1)]^{20}}{[(x-2)(x-2)(x+4)]^{10}} = \lim_{x\to 2}\frac{(x-2)^{20}(x+1)^{20}}{(x-2)^{20}(x+4)^{10}}$$

$$= \lim_{x\to 2}\frac{(x+1)^{20}}{(x+4)^{10}} = \frac{3^{20}}{6^{10}} = \frac{3^{10}\cdot 3^{10}}{3^{10}\cdot 2^{10}} = \left(\frac{3}{2}\right)^{10}$$

(15) $$\lim_{x\to 1}\left(\frac{x^{11}+2x-3}{x^7+x^2-x-1}\right) = \lim_{x\to 1}\left(\frac{x^{11}-1+2x-2}{x^7-x+x^2-1}\right)$$

$$= \lim_{x\to 1}\frac{(x^{11}-1)+2(x-1)}{x(x^6-1)+(x-1)(x+1)}$$; DESARROLLANDO $x^{11}-1$, x^6-1 POR COCIENTES NOTABLES:

$$= \lim_{x\to 1}\frac{(x-1)(x^{10}+x^9+\cdots+x+1)+2(x-1)}{x(x-1)(x^5+x^4\ldots+1)+(x-1)(x+1)}$$ SIMPLIFICANDO $(x-1)$ EN EL NUMERADOR Y EN EL DENOMINADOR:

$$= \lim_{x\to 1}\left(\frac{x^{10}+x^9+\cdots+x^2+x+3}{x^6+x^5+\cdots+x^2+2x+1}\right) = 13/8$$

(16) $$\lim_{x\to 0} \frac{(1+x)^5-(1+5x)}{x^2+x^5}$$

$$\lim_{x\to 0}\frac{x^5+5x^4+10x^3+10x^2+5x+1-(1+5x)}{x^2+x^5}$$

$$\lim_{x\to 0}\frac{x^2(x^3+5x^2+10x+10)}{x^2(x^3+1)} = 10$$

(17) $$\lim_{x\to 1}\left(\frac{x^{60}-2x^{30}+1}{x^{30}-2x^{15}+1}\right)$$

, EL NUMERADOR Y DENOMINADOR SON TRINOMIOS DE LA FORMA $(x^n-1)^2$

$$\lim_{x\to 1}\frac{(x^{30}-1)^2}{(x^{15}-1)^2} = \lim_{x\to 1}\frac{(x^{15}-1)^2(x^{15}+1)^2}{(x^{15}-1)^2} = 2^2 = 4$$

(18) $$\lim_{x\to 1}\frac{(x^n-1)}{(x-1)}, \quad n\in\mathbb{N}$$

, DESARROLLANDO EL NUMERADOR POR COCIENTES NOTABLES:

$$\lim_{x\to 1}\frac{(x-1)(x^{n-1}+x^{n-2}+\cdots+x+1)}{(x-1)} = \overbrace{1+1+\cdots+1}^{\text{"n" veces}} = n$$

(19) $$\lim_{x\to 1}\left(\frac{x^{2n}+x^{-2n}-2}{x^{2n}-x^{-2n}}\right) = \lim_{x\to 1}\left(\frac{x^{2n}-2+\frac{1}{x^{2n}}}{x^{2n}-\frac{1}{x^{2n}}}\right)$$

$$=\lim_{x\to 1}\left(\frac{x^{4n}-2x^{2n}+1}{x^{4n}-1}\right) = \lim_{x\to 1}\frac{(x^{2n}-1)(x^{2n}-1)}{(x^{2n}-1)(x^{2n}+1)} = \frac{0}{2} = 0$$

(20) $$\lim_{x\to 1}\left(\frac{x^n-1}{x^m-1}\right) \quad n,m\in\mathbb{N}$$

(VER SOLUCION DEL PROBLEMA N° 18)

$$\lim_{x\to 1}\frac{(x-1)(x^{n-1}+x^{n-2}+\cdots+x+1)}{(x-1)(x^{m-1}+x^{m-2}+\cdots+x+1)} = \lim_{x\to 1}\frac{x^{n-1}+x^{n-2}+\cdots 1}{x^{m-1}+x^{m-2}+\cdots 1} = \frac{n}{m}$$

(21) $$\lim_{x\to 1}\frac{(x+x^2+x^3+\cdots+x^n-n)}{(x-1)}$$

; ORDENANDO EL NUMERADOR:

$$\lim_{x\to 1}\frac{(x^n+x^{n-1}+\cdots+x^2+x-n)}{(x-1)}$$

FACTORIZANDO EL NUMERADOR

$$\begin{array}{c|cccccc} & 1 & 1 & 1 & \cdots & 1 & -n \\ 1 & & 1 & 2 & \cdots & n-1 & n \\ \hline & 1 & 2 & 3 & & n & 0 \end{array}$$

$$(x-1)(x^{n-1}+2(x^{n-2})+3(x^{n-3})+\cdots(n-1)x+n)$$

RETOMANDO:

$$\lim_{x\to 1}\frac{(x-1)(x^{n-1}+2(x^{n-2})+\cdots+(n-1)x+n)}{(x-1)}$$

$$= 1+2(1)+3(1)+\cdots+(n-1)1+n$$

$$= 1+2+3\cdots+n = \frac{n(n+1)}{2}$$

(22) $\lim\limits_{h\to 0}\dfrac{(x+h)^n - x^n}{h}$

* POR EL TEOREMA DEL BINOMIO DE NEWTON:

$$(x+h)^n = C_0^n x^n + C_1^n x^{n-1}h + C_2^n x^{n-2}h^2 + \cdots + C_n^n h^n$$

$$(x+h)^n = x^n + C_1^n x^{n-1}h + C_2^n x^{n-2}h^2 + \cdots + h^n$$

RETOMANDO:

$$\lim_{h\to 0}\left(\frac{x^n + C_1^n x^{n-1}h + C_2^n x^{n-2}h^2 + \cdots + h^n - x^n}{h}\right)$$

$$\lim_{h\to 0}\frac{h\left(C_1^n x^{n-1} + C_2^n x^{n-2}h + \cdots + h^{n-1}\right)}{h} = C_1^n x^{n-1} = n x^{n-1}$$

* POR COCIENTES NOTABLES:

$$\lim_{h\to 0}\frac{((x+h)-x)\left((x+h)^{n-1} + (x+h)^{n-2}x + (x+h)^{n-3}x^2 + \cdots + x^{n-1}\right)}{h}$$

$$= \lim_{h\to 0}\frac{h\left((x+h)^{n-1} + (x+h)^{n-2}x + \cdots + x^{n-1}\right)}{h}$$

$= (x+h)^{n-1} + (x+h)^{n-1} + \cdots + x^{n-1}$ PARA $h = 0$

$$x^{n-1} + x^{n-1} + \cdots + x^{n-1} = n x^{n-1}$$

(23) $\lim\limits_{x\to 0}\dfrac{(1+mx)^n - (1+nx)^m}{x^2}$

* POR EL TEOREMA DEL BINOMIO DE NEWTON:

$$(1+mx)^n = C_0^n + C_1^n mx + C_2^n (mx)^2 + \cdots + C_n^n (mx)^n$$

$$= 1 + nmx + C_2^n (mx)^2 + \cdots + (mx)^n$$

$$(1+nx)^m = C_0^m + C_1^m nx + C_2^m (nx)^2 + \cdots + C_m^m (nx)^m$$

$$= 1 + mnx + C_2^m (nx)^2 + \cdots + (nx)^m$$

RETOMANDO:

$$\lim_{x\to 0}\frac{1 + nmx + C_2^n (mx)^2 + \cdots (mx)^n - 1 - mnx - C_2^m (nx)^2 - \cdots - (nx)^m}{x^2}$$

SIMPLIFICANDO Y FACTORIZANDO EL NUMERADOR

$$\lim_{x\to 0}\frac{x^2\left(C_2^n m^2 + C_3^n m^3 x + \cdots m^n x^{n-2} - C_2^m n^2 - C_3^m n^3 x - \cdots n^m x^{m-2}\right)}{x^2}$$

FINALMENTE: $C_2^n m^2 - C_2^m n^2$

$$= \frac{n!\, m^2}{2(n-2)!} - \frac{m!\, n^2}{2(m-2)!} = \frac{mn(n-m)}{2}$$

(24) $\lim_{x\to 1}\left(\frac{x^{100}-2x+1}{x^{50}-2x+1}\right) = \lim_{x\to 1}\frac{x^{100}-x-(x-1)}{x^{50}-x-(x-1)}$

$= \lim_{x\to 1}\frac{x(x^{99}-1)-(x-1)}{x(x^{49}-1)-(x-1)}$; DESARROLLANDO POR COCIENTES NOTABLES:

$= \lim_{x\to 1}\frac{x(x-1)(x^{98}+x^{97}+\cdots+1)-(x-1)}{x(x-1)(x^{48}+x^{47}+\cdots+1)-(x-1)}$

$= \lim_{x\to 1}\frac{(x-1)(x^{99}+x^{98}+\cdots+x^2+x)-(x-1)}{(x-1)(x^{49}+x^{48}+\cdots+x^2+x)-(x-1)}$

$= \lim_{x\to 1}\frac{x^{99}+x^{98}+\cdots+x^2+x-1}{x^{49}+x^{48}+\cdots+x^2+x-1} = \frac{99-1}{49-1} = \frac{98}{48} = \frac{49}{24}$

(25) DADAS LAS FUNCIONES $f(x)=\frac{x^3-x^2+2x-2}{x^2-2x+1}$,

$g(x) = \frac{3-4x+2x^2-x^3}{1-2x+x^2}$

(a) PROBAR QUE: $\lim_{x\to 1} f(x)$ Y $\lim_{x\to 1} g(x)$ NO EXISTEN

(b) PROBAR QUE: $\lim_{x\to 1}[f(x)+g(x)]$ EXISTE Y ES IGUAL A 1

* $\lim_{x\to 1} f(x) = \lim_{x\to 1}\left(\frac{x^3-x^2+2x-2}{x^2-2x+1}\right) = \lim_{x\to 1}\frac{(x-1)(x^2+2)}{(x-1)(x-1)}$

$= \frac{3}{0}$ (NO EXISTE)

* $\lim_{x\to 1} g(x) = \lim_{x\to 1}\left(\frac{3-4x+2x^2-x^3}{1-2x+x^2}\right) = -\lim\frac{(x-1)(3x^2-x+1)}{(x-1)(x-1)}$

$= \frac{3}{0}$ (NO EXISTE)

* $\lim_{x\to 1}[f(x)+g(x)] = \lim_{x\to 1}\left(\frac{x^3-x^2+2x-2}{x^2-2x+1}+\frac{3-4x+2x^2-x^3}{1-2x+x^2}\right)$

$= \lim_{x\to 1}\left(\frac{x^2-2x+1}{x^2-2x+1}\right) = \lim_{x\to 1} 1 = 1$ (L.q.q.d)

(26) $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9}\frac{(\sqrt{x}-3)}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim_{x\to 9}\left(\frac{1}{\sqrt{x}+3}\right) = \frac{1}{6}$

(27) $\lim_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49} = \lim_{x\to 7}\frac{(2-\sqrt{x-3})(2+\sqrt{x-3})}{(x-7)(x+7)(2+\sqrt{x-3})}$

$= -\lim_{x\to 7}\frac{(x-7)}{(x-7)(x+7)(2+\sqrt{x-3})} = -\frac{1}{56}$

(28) $\lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} = \lim_{x\to 1}\frac{(\sqrt{x}-1)(\sqrt{x}+1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}{(\sqrt[3]{x}-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)(\sqrt{x}+1)}$

$= \lim_{x\to 1}\frac{(x-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1}\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt{x}+1} = \frac{3}{2}$

(29) $\lim\limits_{x\to5} \dfrac{4-\sqrt{11+x}}{2-\sqrt{9-x}}$

$$\lim_{x\to5}\frac{(4-\sqrt{11+x})(4+\sqrt{11+x})(2+\sqrt{9-x})}{(2-\sqrt{9-x})(2+\sqrt{9-x})(4+\sqrt{11+x})}$$

$$= -\lim_{x\to5}\frac{(x-5)(2+\sqrt{9-x})}{(x-5)(4+\sqrt{11+x})} = -\lim_{x\to5}\frac{(2+\sqrt{9-x})}{(4+\sqrt{11+x})} = -\frac{1}{2}$$

(30) $\lim\limits_{x\to3} \dfrac{\sqrt{x+13}-2\sqrt{x+1}}{x^2-9}$

$$\lim_{x\to3}\frac{(\sqrt{x+13}-\sqrt{4x+4})(\sqrt{x+13}+\sqrt{4x+4})}{(x-3)(x+3)(\sqrt{x+13}+\sqrt{4x+4})}$$

$$= -\lim_{x\to3}\frac{3(x-3)}{(x-3)(\sqrt{x+13}+\sqrt{4x+4})} = -3\lim_{x\to3}\frac{1}{\sqrt{x+13}+\sqrt{4x+4}}$$

$$= -\frac{1}{16}$$

(31) $\lim\limits_{x\to0} \dfrac{\sqrt[3]{a+x}-\sqrt[3]{a}}{x} = \lim\limits_{x\to0}\dfrac{(\sqrt[3]{a+x}-\sqrt[3]{a})(\sqrt[3]{a+x}^{\,2}+\sqrt[3]{(a+x)a}+\sqrt[3]{a^2})}{x(\sqrt[3]{a+x}^{\,2}+\sqrt[3]{(a+x)a}+\sqrt[3]{a^2})}$

$$= \lim_{x\to0}\left(\frac{1}{\sqrt[3]{a+x}^{\,2}+\sqrt[3]{(a+x)a}+\sqrt[3]{a^2}}\right) = \frac{1}{3}a^{-2/3}$$

(32) $\lim\limits_{x\to8}\left(\dfrac{\sqrt{2x}-4}{\sqrt[3]{x}-2}\right) = \lim\limits_{x\to8}\dfrac{(\sqrt{2x}-4)(\sqrt{2x}+4)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}{(\sqrt[3]{x}-2)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)(\sqrt{2x}+4)}$

$$= 2\lim_{x\to8}\left(\frac{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}{\sqrt{2x}+4}\right) = \frac{12(2)}{8} = 3$$

(33) $\lim\limits_{x\to-8}\left(\dfrac{\sqrt{1-x}-3}{2+\sqrt[3]{x}}\right) = \lim\limits_{x\to-8}\dfrac{(\sqrt{1-x}-3)(\sqrt{1-x}+3)(4-2\sqrt[3]{x}+\sqrt[3]{x^2})}{(2+\sqrt[3]{x})(4-2\sqrt[3]{x}+\sqrt[3]{x^2})(\sqrt{1-x}+3)}$

$$= -\lim_{x\to-8}\left(\frac{4-2\sqrt[3]{x}+\sqrt[3]{x^2}}{\sqrt{1-x}+3}\right) = -2$$

(34) $\lim\limits_{x\to0}\dfrac{\sqrt{x^2+2ax+b}-\sqrt{x^2-2ax+b}}{\sqrt{2x+c}-\sqrt{c-2x}}$

$$= \lim_{x\to0}\frac{(\sqrt{x^2+2ax+b}-\sqrt{x^2-2ax+b})(\sqrt{x^2+2ax+b}+\sqrt{x^2-2ax+b})}{(\sqrt{2x+c}-\sqrt{c-2x})(\sqrt{x^2+2ax+b}+\sqrt{x^2-2ax+b})}$$

$$= \lim_{x\to0}\frac{4ax(\sqrt{2x+c}+\sqrt{c-2x})}{4x(\sqrt{x^2+2ax+b}+\sqrt{x^2-2ax+b})} = \frac{2\sqrt{c}\,a}{2\sqrt{b}} = a\sqrt{\frac{c}{b}}$$

(35) $\lim\limits_{x\to a}\left(\dfrac{\sqrt[5]{x}-\sqrt[5]{a}}{x-a}\right) = \lim\limits_{x\to a}\dfrac{(\sqrt[5]{x}-\sqrt[5]{a})(\sqrt[5]{x^4}+\sqrt[5]{x^3a}+\sqrt[5]{x^2a^2}+\sqrt[5]{xa^3}+\sqrt[5]{a^4})}{(x-a)(\sqrt[5]{x^4}+\sqrt[5]{x^3a}+\sqrt[5]{x^2a^2}+\sqrt[5]{xa^3}+\sqrt[5]{a^4})}$

$$= \lim_{x\to a}\left(\frac{1}{\sqrt[5]{x^4}+\sqrt[5]{x^3a}+\sqrt[5]{x^2a^2}+\sqrt[5]{xa^3}+\sqrt[5]{a^4}}\right) = \frac{1}{5}a^{-4/5}$$

36 $$\lim_{x\to a}\left(\frac{\sqrt[n]{x}-\sqrt[n]{a}}{x-a}\right)=\lim_{x\to a}\frac{(\sqrt[n]{x}-\sqrt[n]{a})(\sqrt[n]{x^{n-1}}+\sqrt[n]{x^{n-2}a}+\cdots\sqrt[n]{a^{n-1}})}{(x-a)(\sqrt[n]{x^{n-1}}+\sqrt[n]{x^{n-2}a}+\cdots\sqrt[n]{a^{n-1}})}$$

$$=\lim_{x\to a}\frac{(x-a)}{(x-a)(\sqrt[n]{x^{n-1}}+\sqrt[n]{x^{n-2}a}+\cdots\sqrt[n]{a^{n-1}})}=\frac{1}{n}a^{-\frac{(n-1)}{n}}$$

37 $$\lim_{x\to 8}\left(\frac{\sqrt{7+\sqrt[3]{x}}-3}{x-8}\right)$$

$$=\lim_{x\to 8}\frac{(\sqrt{7+\sqrt[3]{x}}-3)(\sqrt{7+\sqrt[3]{x}}+3)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}{(x-8)(\sqrt{7+\sqrt[3]{x}}+3)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}$$

$$=\lim_{x\to 8}\frac{1}{(\sqrt{7+\sqrt[3]{x}}+3)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}=\frac{1}{72}$$

38 $$\lim_{x\to 1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}=\lim_{x\to 1}\left(\frac{\sqrt[3]{x}-1}{x-1}\right)^2$$

$$=\lim_{x\to 1}\frac{(\sqrt[3]{x}-1)^2(\sqrt[3]{x^2}+\sqrt[3]{x}+1)^2}{(x-1)^2(\sqrt[3]{x^2}+\sqrt[3]{x}+1)^2}=\lim_{x\to 1}\left(\frac{1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}\right)^2=\frac{1}{9}$$

39 $$\lim_{x\to 0}\frac{\sqrt[4]{x+1}-\sqrt[3]{x+1}}{x}$$, HACIENDO: $x+1=Y^{12}$

SI $x\to 0$, $\therefore Y\to 1$

$$=\lim_{Y\to 1}\left(\frac{\sqrt[4]{Y^{12}}-\sqrt[3]{Y^{12}}}{Y^{12}-1}\right)=-\lim_{Y\to 1}\frac{Y^3(Y-1)}{(Y^6+1)(Y^3+1)(Y-1)(Y^2+Y+1)}$$

$$=-\frac{1}{12}$$

40 $$\lim_{x\to 0}\left(\frac{\sqrt[4]{x^4+1}-\sqrt{x^3+1}}{x^3}\right)$$

$$=\lim_{x\to 0}\frac{(\sqrt[4]{x^4+1}-\sqrt{x^3+1})(\sqrt[4]{x^4+1}+\sqrt{x^3+1})(\sqrt{x^4+1}+x^3+1)}{x^3(\sqrt[4]{x^4+1}+\sqrt{x^3+1})(\sqrt{x^4+1}+x^3+1)}$$

$$=\lim_{x\to 0}\frac{x^3(x-x^3-2)}{x^3(\sqrt[4]{x^4+1}+\sqrt{x^3+1})(\sqrt{x^4+1}+x^3+1)}=-\frac{1}{2}$$

41 $$\lim_{x\to 3}\left(\frac{\sqrt{x}-3+x-\sqrt{3}}{x-3}\right)=\lim_{x\to 3}\left(\frac{\sqrt{x}-\sqrt{3}}{x-3}+\frac{x-3}{x-3}\right)$$

$$=\lim_{x\to 3}\left[\frac{(\sqrt{x}-\sqrt{3})}{(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})}+1\right]=\lim_{x\to 3}\left(\frac{1}{\sqrt{x}+\sqrt{3}}+1\right)=\frac{1}{2\sqrt{3}}+1$$

42 $$\lim_{x\to 1}\left(\frac{x\sqrt{x}-x+\sqrt{x}-1}{x-1}\right)=\lim_{x\to 1}\frac{x(\sqrt{x}-1)+(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$=\lim_{x\to 1}\left(\frac{x+1}{\sqrt{x}+1}\right)=1$$

43 $$\lim_{x\to 0}\frac{\sqrt[3]{8+3x-x^2}-2}{x+x^2}$$

$$=\lim_{x\to 0}\frac{(\sqrt[3]{8+3x-x^2}-2)(\sqrt[3]{8+3x-x^2}^{\,2}+2\sqrt[3]{8+3x-x^2}+4)}{(x+x^2)(\sqrt[3]{8+3x-x^2}^{\,2}+2\sqrt[3]{8+3x-x^2}+4)}$$

$$= \lim_{x\to 0} \frac{(3-x)}{(x+1)\left(\sqrt[3]{8+3x-x^2}^{\,2}+2\sqrt[3]{8+3x-x^2}+4\right)} = \frac{1}{4}$$

(44) $\displaystyle \lim_{x\to 0} \frac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{x+2\sqrt[3]{x^4}}$; RACIONALIZANDO EL NUMERADOR:

$$= \lim_{x\to 0} \frac{(\sqrt[3]{27+x}-\sqrt[3]{27-x})(\sqrt[3]{27+x}^{\,2}+\sqrt[3]{27^2-x^2}+\sqrt[3]{27-x}^{\,2})}{x(1+2\sqrt[3]{x})(\sqrt[3]{27+x}^{\,2}+\sqrt[3]{27^2-x^2}+\sqrt[3]{27-x}^{\,2})}$$

$$= \lim_{x\to 0} \frac{2}{(1+2\sqrt[3]{x})(\sqrt[3]{27+x}^{\,2}+\sqrt[3]{27^2-x^2}+\sqrt[3]{27-x}^{\,2})} = \frac{2}{27}$$

(45) $\displaystyle \lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}$; RACIONALIZANDO EL NUMERADOR I EL DENOMINADOR:

$$= \lim_{x\to 0} \frac{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})(\sqrt[3]{1+x}^{\,2}+\sqrt[3]{1-x^2}+\sqrt[3]{1-x}^{\,2})}{(\sqrt[3]{1+x}-\sqrt[3]{1-x})(\sqrt[3]{1+x}^{\,2}+\sqrt[3]{1-x^2}+\sqrt[3]{1-x}^{\,2})(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \lim_{x\to 0} \frac{\sqrt[3]{1+x}^{\,2}+\sqrt[3]{1-x^2}+\sqrt[3]{1-x}^{\,2}}{\sqrt{1+x}+\sqrt{1-x}} = \frac{3}{2}$$

(46) $\displaystyle \lim_{x\to 0} \frac{x^2}{\sqrt[5]{1+5x}-(1+x)}$; RACIONALIZANDO EL DENOMINADOR:

$$= \lim_{x\to 0} \frac{x^2\left(\sqrt[5]{1+5x}^{\,4}+\sqrt[5]{1+5x}^{\,3}(1+x)+\sqrt[5]{1+5x}^{\,2}(1+x)^2+\sqrt[5]{1+5x}(1+x)^3+(1+x)^4\right)}{1+5x-1-5x-10x^2-10x^3-5x^4-x^5}$$

FINALMENTE:

$$= -\lim_{x\to 0} \frac{\sqrt[5]{1+5x}^{\,4}+\sqrt[5]{1+5x}^{\,3}(1+x)+\sqrt[5]{1+5x}^{\,2}(1+x)^2+\sqrt[5]{1+5x}(1+x)^3+(1+x)^4}{10+10x+5x^2+x^3}$$

$$= -\frac{5}{10} = -\frac{1}{2}$$

(47) $\displaystyle \lim_{x\to 7} \frac{\sqrt{x+2}-\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2}$; SUMAMOS I RESTAMOS 3 EN EL NUMERADOR

$$= \lim_{x\to 7} \frac{\sqrt{x+2}-3-\sqrt[3]{x+20}-3}{\sqrt[4]{x+9}-2}$$

$$= \lim_{x\to 7} \frac{\sqrt{x+2}-3}{\sqrt[4]{x+9}-2} - \lim_{x\to 7} \frac{\sqrt[3]{x+20}-3}{\sqrt[4]{x+9}-2}$$; RESOLVEMOS POR SEPARADO AMBOS LIMITES:

$$* \lim_{x\to 7} \frac{(x-7)(\sqrt[4]{x+9}^{\,3}+2\sqrt[4]{x+9}^{\,2}+4\sqrt[4]{x+9}+8)}{(x-7)(\sqrt{x+2}+3)} = \frac{16}{3}$$

$$* \lim_{x\to 7} \frac{(x-7)(\sqrt[4]{x+9}^{\,3}+2\sqrt[4]{x+9}^{\,2}+4\sqrt[4]{x+9}+8)}{(x-7)(\sqrt[3]{x+20}^{\,2}+3\sqrt[3]{x+20}+9)} = \frac{32}{27}$$

FINALMENTE:

$$\frac{16}{3} - \frac{32}{27} = \frac{112}{27}$$

* HALLAMOS $P(x)$:

$$\frac{P(x)}{(x-a)(x-b)(x-c)} = \frac{A(x-b)(x-c)+B(x-a)(x-c)+C(x-a)(x-b)}{(x-a)(x-b)(x-c)}$$

* $\displaystyle \lim_{x\to a} \frac{P(x)}{(x-b)(x-c)} = \lim_{x\to a}\left(A+\frac{B(x-a)}{(x-b)}+\frac{C(x-a)}{(x-c)}\right) = A$

* $\displaystyle \lim_{x\to b} \frac{P(x)}{(x-a)(x-c)} = \lim_{x\to b}\left(\frac{A(x-b)}{(x-a)}+B+\frac{C(x-b)}{(x-c)}\right) = B$

* $\displaystyle \lim_{x\to c} \frac{P(x)}{(x-a)(x-b)} = \lim_{x\to c}\left(\frac{A(x-c)}{(x-a)}+\frac{B(x-c)}{(x-b)}+C\right) = C$

(b) DESCOMPONER EN FRACCIONES PARCIALES:

$$\frac{3x^2+11x-52}{(x-3)(x+1)(x-2)}$$

$$\frac{3x^2+11x-52}{(x-3)(x+1)(x-2)} = \frac{A}{(x-3)}+\frac{B}{(x+1)}+\frac{C}{(x-2)}$$

$$3x^2+11x-52 = (A+B+C)x^2-(A+5B+2C)x-2A+6B-3C$$

$$\left.\begin{aligned} A+B+C &= 3 \\ A+5B+2C &= -11 \\ 2A-6B+3C &= 52 \end{aligned}\right\} \quad \begin{aligned} A&=2 \\ B&=-5 \\ C&=6 \end{aligned} \quad \therefore \quad \frac{2}{x-3}-\frac{5}{x+1}+\frac{6}{x-2}$$

Limites Unilaterales

CALCULAR LOS LIMITES QUE SE INDICAN:

① (a) $f(x)=\begin{cases} 2x+1, & x<3 \\ 10-x, & x\geq 3 \end{cases}$; CUANDO $x\to 3^+,\ x\to 3^-,\ x\to 3$

* $\displaystyle \lim_{x\to 3^+} f(x),\ x>3 \ \therefore \ \lim_{x\to 3^+}(10-x) = 7$

* $\displaystyle \lim_{x\to 3^-} f(x),\ x<3 \ \therefore \ \lim_{x\to 3^-}(2x+1) = 7$

* $\displaystyle \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = 7 \ \therefore \ \lim_{x\to 3} f(x) = 7$

(b) $f(x)=\begin{cases} 4x+1, & x<2 \\ 6-x, & x\geq 2 \end{cases}$; CUANDO $x\to 2^-,\ x\to 2^+,\ x\to 2$

* $\displaystyle \lim_{x\to 2^-} f(x),\ x<2 \ \therefore \ \lim_{x\to 2^-}(4x+1) = 9$

* $\displaystyle \lim_{x\to 2^+} f(x),\ x>2 \ \therefore \ \lim_{x\to 2^+}(6-x) = 4$

* $\displaystyle \lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x) \ \therefore \ \lim_{x\to 2} f(x)$ NO EXISTE

② (a) $f(x)=\begin{cases} 2x+3, & x<1 \\ 2, & x=1 \\ 7-2x, & x>1 \end{cases}$; CUANDO: $x\to 1^+,\ x\to 1^-,\ x\to 1$

* $\displaystyle \lim_{x\to 1^+} f(x);\ x>1 \ \therefore \ \lim_{x\to 1^+}(7-2x) = 5$

* $\lim\limits_{x\to 1^-} f(x)$, $x<1$ $\therefore$ $\lim\limits_{x\to 1^-} (2x+3) = 5$

* $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^-} f(x) = 5$ $\therefore$ $\lim\limits_{x\to 1} f(x) = 5$

(b) $f(x) = \begin{cases} x^2, & x<1 \\ x, & 1<x<4 \\ 4-x, & x>4 \end{cases}$ CUANDO: $x\to 1$, $x\to 4$

$$*\lim_{x\to 1} f(x) \begin{cases} \lim\limits_{x\to 1^+} f(x), \ x>1 \ \therefore \ \lim\limits_{x\to 1^+} x = 1 \\ \lim\limits_{x\to 1^-} f(x), \ x<1 \ \therefore \ \lim\limits_{x\to 1^-} x^2 = 1 \end{cases}$$

$\Rightarrow \lim\limits_{x\to 1} f(x) = 1$

$$*\lim_{x\to 4} f(x) \begin{cases} \lim\limits_{x\to 4^+} f(x), \ x>4 \ \therefore \ \lim\limits_{x\to 4^+} (4-x) = 0 \\ \lim\limits_{x\to 4^-} f(x), \ x<4 \ \therefore \ \lim\limits_{x\to 4^-} x = 4 \end{cases}$$

$\Rightarrow \lim\limits_{x\to 4} f(x)$ NO EXISTE

③ $f(x) = \begin{cases} 1/(1-x) & , x<0 \\ 0 & , x=0 \\ x & , 0<x<1 \\ 2 & , x\geq 1 \end{cases}$ CUANDO: $x\to 0^-, x\to 0^+, x\to 1^-, x\to 1^+$

* $\lim\limits_{x\to 0^-} f(x)$, $x<0$ $\therefore$ $\lim\limits_{x\to 0^-} \dfrac{1}{(1-x)} = 1$

* $\lim\limits_{x\to 0^+} f(x)$, $x>0$ $\therefore$ $\lim\limits_{x\to 0^+} x = 0$

* $\lim\limits_{x\to 1^-} f(x)$, $x<1$ $\therefore$ $\lim\limits_{x\to 1^-} x = 1$

* $\lim\limits_{x\to 1^+} f(x)$, $x>1$ $\therefore$ $\lim\limits_{x\to 1^+} 2 = 2$

④ $f(x) = \begin{cases} 1/(x-2) & , x<1 \\ 0 & , x=1 \\ x-2 & , 1<x<2 \\ 2 & , x\geq 2 \end{cases}$ CUANDO: $x\to 1$, $x\to 2$

$$*\lim_{x\to 1} f(x) \begin{cases} \lim\limits_{x\to 1^+} f(x), \ x>1 \ \therefore \ \lim\limits_{x\to 1^+} (x-2) = -1 \\ \lim\limits_{x\to 1^-} f(x), \ x<1 \ \therefore \ \lim\limits_{x\to 1^-} \dfrac{1}{(x-2)} = -1 \end{cases}$$

$\Rightarrow \lim\limits_{x\to 1} f(x) = -1$

$$*\lim_{x\to 2} f(x) \begin{cases} \lim\limits_{x\to 2^+} f(x), \ x>2 \ \therefore \ \lim\limits_{x\to 2^+} 2 = 2 \\ \lim\limits_{x\to 2^-} f(x), \ x<2 \ \therefore \ \lim\limits_{x\to 2^-} (x-2) = 0 \end{cases}$$

$\Rightarrow \lim\limits_{x\to 2} f(x)$ NO EXISTE

⑤ $f(x) = \dfrac{x-1}{|x-1|}$, CUANDO $x\to 1^+$, $x\to 1^-$

* $\lim\limits_{x\to 1^+} \dfrac{x-1}{|x-1|}$, $x>1$ ó $x-1>0$ $\Rightarrow |x-1| = x-1$ $\therefore$ $\lim\limits_{x\to 1^+} \dfrac{x-1}{x-1} = 1$

$$* \lim_{x\to 1^-} \frac{x-1}{|x-1|}, \quad x<1 \text{ ó } x-1<0 \;\therefore\; \lim_{x\to 1^-} \frac{(x-1)}{-(x-1)}$$

$$\Rightarrow |x-1| = -(x-1)$$

$$= \lim_{x\to 1^-} (-1) = -1$$

⑥ $f(x) = \dfrac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}$ $(a>0)$ CUANDO: $x\to a^+, x\to a^-$

HACIENDO PREVIAS TRANSFORMACIONES:

$$f(x) = \frac{\sqrt{x}-\sqrt{a}}{\sqrt{x^2-a^2}} + \frac{\sqrt{x-a}}{\sqrt{x^2-a^2}}, \quad f(x) = \frac{(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a})\sqrt{x^2-a^2}}{\sqrt{x^2-a^2}\sqrt{x^2-a^2}(\sqrt{x}+\sqrt{a})} + \frac{\sqrt{x-a}}{\sqrt{x-a}\sqrt{x+a}}$$

$$f(x) = \frac{(x-a)\sqrt{x^2-a^2}}{(x^2-a^2)(\sqrt{x}+\sqrt{a})} + \frac{1}{\sqrt{x+a}}, \quad f(x) = \frac{\sqrt{x^2-a^2}}{(x+a)(\sqrt{x}+\sqrt{a})} + \frac{1}{\sqrt{x+a}}$$

RETOMANDO:

$$* \lim_{x\to a^+}\left[\frac{\sqrt{x^2-a^2}}{(x+a)(\sqrt{x}+\sqrt{a})} + \frac{1}{\sqrt{x+a}}\right] \quad x>a \text{ ó } x-a>0$$

$$= \frac{0}{2a\sqrt{a}} + \frac{1}{\sqrt{2a}} = \frac{1}{\sqrt{2a}}$$

$$* \lim_{x\to a^-}\left[\frac{\sqrt{x^2-a^2}}{(x+a)(\sqrt{x}+\sqrt{a})} + \frac{1}{\sqrt{x+a}}\right] \quad x<a \text{ ó } x-a<0$$

$\therefore \sqrt{x-a}$ NO EXISTE

$$\therefore \lim_{x\to a^-}\left[\frac{\sqrt{x-a}\sqrt{x+a}}{(x+a)(\sqrt{x}+\sqrt{a})} + \frac{1}{\sqrt{x+a}}\right] \text{ NO EXISTE}$$

⑦ $f(x) = \dfrac{x^2+x}{|x|}$ $x\neq 0$ CUANDO $x\to 0^+, x\to 0^-$

$$* \lim_{x\to 0^+} \frac{x^2+x}{|x|}, \quad x>0 \;\therefore |x| = x \Rightarrow \lim_{x\to 0^+} \frac{x(x+1)}{x} = 1$$

$$* \lim_{x\to 0^-} \frac{x^2+x}{|x|}, \quad x<0 \;\therefore |x| = -x \Rightarrow \lim_{x\to 0^-} \frac{x(x+1)}{-x} = -1$$

⑧ $f(x) = \dfrac{|x|-x}{x}$, CUANDO $x\to 0^-, x\to 0^+, x\to 0$

$$* \lim_{x\to 0^-} \frac{|x|-x}{x}, \quad x<0 \;\therefore |x| = -x \Rightarrow \lim_{x\to 0^-} \frac{-2x}{x} = -2$$

$$* \lim_{x\to 0^+} \frac{|x|-x}{x}, \quad x>0 \;\therefore |x| = x \Rightarrow \lim_{x\to 0^+} \frac{0}{x} = \lim_{x\to 0^+} 0 = 0$$

$$* \lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x) \;\therefore \lim_{x\to 0} f(x) \text{ NO EXISTE}$$

⑨ $f(x) = |x| + |x-1|$, SI $x\to 0$, $x\to 1$

$$* \lim_{x\to 0} f(x) \begin{cases} \lim_{x\to 0^+} f(x) \quad x>0 \;\therefore |x| = x,\ |x-1| = -(x-1), \text{ PORQUE } x\to 0 \\ \lim_{x\to 0^-} f(x) \quad x<0 \;\therefore |x| = -x,\ |x-1| = -(x-1) \end{cases}$$

$$* \lim_{x\to 0^+} (|x|+|x-1|) = \lim_{x\to 0^+} (x-(x-1)) = \lim_{x\to 0^+} 1 = 1$$

$$* \lim_{x\to 0^-} (|x|+|x-1|) = \lim_{x\to 0^-} (-x-x+1) = \lim_{x\to 0^-} (1-2x) = 1$$

$$\therefore \lim_{x\to 0} f(x) = 1$$

Limites al Infinito

Calcular los siguientes limites:

$$\lim_{x\to\infty}\left(\frac{3x+5}{x+1}\right)=\lim_{x\to\infty}\left(\frac{3+\frac{5}{x}}{1+\frac{1}{x}}\right)=\frac{3+0}{1+0}=3$$

$$\lim_{x\to-\infty}\left(\frac{3x+5}{x+1}\right)=\lim_{x\to-\infty}\left(\frac{3+\frac{5}{x}}{1+\frac{1}{x}}\right)=\frac{3+0}{1+0}=3$$

$$\lim_{x\to+\infty}\left(\frac{x^2-2x+5}{7x^3+x+1}\right)=\lim_{x\to+\infty}\left(\frac{\frac{x^2}{x^3}-\frac{2x}{x^3}+\frac{5}{x^3}}{\frac{7x^3}{x^3}+\frac{x}{x^3}+\frac{1}{x^3}}\right)$$

$$=\lim_{x\to+\infty}\left(\frac{\frac{1}{x}-\frac{2}{x}+\frac{5}{x^3}}{7+\frac{1}{x^2}+\frac{1}{x^3}}\right)=\frac{0}{7}=0$$

$$\lim_{x\to-\infty}\left(\frac{x^2-2x+5}{7x^3+x+1}\right)=\lim_{x\to-\infty}\left(\frac{\frac{1}{x}-\frac{2}{x^2}+\frac{5}{x^3}}{7+\frac{1}{x^2}+\frac{1}{x^2}}\right)=0$$

$$\lim_{x\to\infty}\left(\frac{2x^2-x+3}{x^3-8x+5}\right)=\lim_{x\to\infty}\left(\frac{\frac{2}{x}-\frac{1}{x^2}+\frac{3}{x^3}}{1-\frac{8}{x^2}+\frac{5}{x^3}}\right)=\frac{0}{1}=0$$

$$\lim_{x\to\infty}\left(\frac{x^3-8x+5}{2x^2-x+3}\right)=\lim_{x\to\infty}\left(\frac{1-\frac{8}{x^2}+\frac{5}{x^3}}{\frac{2}{x}-\frac{1}{x^2}+\frac{3}{x^3}}\right)=\frac{1}{0}=\infty$$

$$\lim_{x\to\infty}\left(\frac{x^2+a^2}{x^3+a^3}\right)=\lim_{x\to\infty}\left(\frac{\frac{1}{x}+\frac{a^2}{x^3}}{1+\frac{a^3}{x^3}}\right)=\frac{0}{1}=0$$

$$\lim_{x\to\infty}\left(\frac{2x^3+x^2-3}{x^3+x+2}\right)=\left(\frac{2+\frac{1}{x}-\frac{3}{x^3}}{1+\frac{a^3}{x^3}}\right)=2$$

$$\lim_{x\to\infty}\frac{(x-1)(x-2)(x-3)(x-4)(x-5)}{(5x-1)^5}$$

$$=\lim_{x\to\infty}=\frac{\left(1-\frac{1}{x}\right)\left(1-\frac{2}{x}\right)\left(1-\frac{3}{x}\right)\left(1-\frac{4}{x}\right)\left(1-\frac{5}{x}\right)}{\left(5-\frac{1}{x}\right)^5}$$

$$=\frac{(1)(1)(1)(1)(1)}{(5)^5}=\frac{1}{5^5}=5^{-5}$$

$$\lim_{x\to\infty}\left(\frac{4x^3+2x^2-5}{8x^3+x+2}\right)=\lim_{x\to\infty}\left(\frac{4+\frac{2}{x}-\frac{5}{x^3}}{8+\frac{1}{x^2}+\frac{2}{x^3}}\right)=\frac{4}{8}=\frac{1}{2}$$

$$\lim_{x\to\infty}\frac{(2x+3)^3(3x-2)^2}{x^5+5}=\lim_{x\to\infty}\frac{\left(2+\frac{3}{x}\right)^3\left(3-\frac{2}{x}\right)^2}{1+\frac{5}{x^5}}=72$$

$$\lim_{x\to\infty}\frac{(2x-3)^{20}(3x+2)^{30}}{(2x+1)^{50}}=\frac{(2x-3)^{20}(3x+2)^{30}}{(2x+1)^{20}(2x+1)^{30}}$$

$$=\lim_{x\to\infty}\left(\frac{2x-3}{2x+1}\right)^{20}\left(\frac{3x+2}{2x+1}\right)^{30}=\lim_{x\to\infty}\left(\frac{2-\frac{3}{x}}{2+\frac{1}{x}}\right)^{20}\left(\frac{3+\frac{2}{x}}{2+\frac{1}{x}}\right)^{30}$$

$$=(1)^{20}\left(\frac{3}{2}\right)^{30}=\left(\frac{3}{2}\right)^{30}$$

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+9}}{x+3}=\lim_{x\to-\infty}\frac{\sqrt{1+\frac{9}{x^2}}}{1+\frac{3}{x}}=\frac{1}{1}=1$$

$$\lim_{x\to\infty}\frac{2x^2-3x+4}{\sqrt{x^4+1}}=\lim_{x\to\infty}\frac{\frac{2x^2}{x^2}-\frac{3x}{x^2}+\frac{4}{x^2}}{\sqrt{\frac{x^4+1}{x^4}}}$$

$$=\lim_{x\to\infty}\left(\frac{2-\frac{3}{x}+\frac{4}{x^2}}{\sqrt{1+\frac{1}{x^4}}}\right)=\frac{2}{1}=2$$

$$\lim_{x\to\infty}\left(\frac{2x+3}{x+\sqrt[3]{x}}\right)=\lim_{x\to\infty}\left(\frac{\frac{2x+3}{x}}{\frac{x+\sqrt[3]{x}}{x}}\right)=\frac{2+\frac{3}{x}}{1+\frac{\sqrt[3]{x}}{x}}=2$$

$$\lim_{x\to\infty}\left(\frac{x^2}{10+x\sqrt{x}}\right)=\lim_{x\to\infty}\left(\frac{1}{\frac{10}{x^2}+\sqrt{\frac{x}{x^2}}}\right)=\frac{1}{0}=\infty$$

$$\lim_{x\to\infty}\frac{\sqrt[3]{x^2+1}}{x+1}=\lim_{x\to\infty}\frac{\sqrt[3]{\frac{x^2+1}{x^3}}}{1+\frac{1}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt[3]{\frac{1}{x}+\frac{1}{x^3}}}{1+\frac{1}{x}}=\frac{0}{1}=0$$

$$\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}=\frac{\frac{\sqrt{x}}{\sqrt{x}}}{\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}}$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}}=1$$

$$\lim_{x\to\infty}\left(\sqrt{x^2-5x+6}-x\right)$$

$$\lim_{x\to+\infty}\frac{\left(\sqrt{x^2-5x+6}-x\right)\left(\sqrt{x^2-5x+6}+x\right)}{\sqrt{x^2-5x+6}+x}$$

$$=\lim_{x\to+\infty}\frac{-5x+6}{\sqrt{x^2-5x+6}+x}=\lim_{x\to\infty}\frac{-5+\frac{6}{x}}{\sqrt{1-\frac{5}{x}+\frac{6}{x^2}}+1}$$

$$=\frac{-5}{\sqrt{1}+1}=\frac{-5}{1+1}=-\frac{5}{2}$$

$$\lim_{x\to+\infty}\left(\sqrt{x+a}-\sqrt{x}\right)=\lim_{x\to\infty}\frac{a}{\sqrt{x+a}+\sqrt{x}}=\frac{a}{\infty}=0$$

$$\lim_{x\to-\infty}\left(x+\sqrt[3]{1-x^3}\right)=\left(\sqrt[3]{x^3}+\sqrt[3]{1-x^3}\right)$$

Sabemos que: $a+b=\left(\sqrt[3]{a}+\sqrt[3]{b}\right)\left(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}\right)$

$$\Rightarrow\lim_{x\to-\infty}\frac{1}{\sqrt[3]{(x^3)^2}+\sqrt[3]{x^3(1-x^3)}+\sqrt[3]{(1-x^3)^2}}$$

$$=\lim_{x\to-\infty}\frac{1}{x^2+x\sqrt[3]{1-x^3}+\sqrt[3]{(1-x^3)^2}}=\frac{1}{-\infty}=0$$

$$\lim_{x\to+\infty}x\left(\sqrt{x^2+1}-x\right)=\lim_{x\to+\infty}=\frac{x}{\sqrt{x^2+1}+x}$$

$$=\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{x^2}}+1}=\frac{1}{1+1}=\frac{1}{2}$$

$$\lim_{x\to+\infty} x\left(\sqrt{x^2+1}-x\right) = \lim_{x\to+\infty}\frac{x}{\sqrt{x^2+1}+x}$$

$$\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{x^2}}+1} = \frac{1}{1+1} = \frac{1}{2}$$

$$\lim_{x\to+\infty}\left(\sqrt{x(x+b)}-x\right) = \lim_{x\to+\infty}\frac{bx}{\sqrt{x(x+b)}+x}$$

$$\lim_{x\to+\infty}\frac{b}{\sqrt{1+\frac{b}{x^2}}+1} = \frac{b}{\sqrt{1+0}+1} = \frac{b}{2}$$

$$\lim_{x\to+\infty}\left(\sqrt{x^2+1}-x\right) = \frac{1}{\sqrt{x^2+1}+x} = \frac{1}{\infty} = 0$$

$$\lim_{x\to-\infty}\left(\sqrt[3]{x^3+x}-\sqrt[3]{x^3+1}\right)$$

$$= \lim_{x\to-\infty}\frac{x-1}{\left(\sqrt[3]{(x^3+x)^2}+\sqrt[3]{(x^3+x)(x^3+1)}+\sqrt[3]{(x^3+1)^2}\right)}$$

$$= \lim_{x\to-\infty}\frac{1-\frac{1}{x}}{F.R} = \frac{1+0}{\infty} = 0$$

$$\lim_{x\to\infty}\sqrt{\frac{(x^3-2x+1)^2}{(2x^2+x-1)^3}} = \lim_{x\to\infty}\frac{x^3-2x+1}{\sqrt{(2x^2+x-1)^3}}$$

$$\lim_{x\to\infty}\frac{1-\frac{2}{x^2}+\frac{1}{x^3}}{\sqrt{\left(2+\frac{1}{x}-\frac{1}{x^2}\right)^3}} = \frac{1}{\sqrt{8}} = \frac{\sqrt{2}}{4}$$

$$\lim_{x\to+\infty}(x^2+1)^5(\sqrt{x}-1)^3\frac{(x^2+1)^2}{(2x^2-5)^2}$$

$$= \lim_{x\to+\infty}(x^2+1)(\sqrt{x}-1)^3\frac{\left(1+\frac{1}{x^2}\right)^2}{\left(2-\frac{5}{x^2}\right)^2} = \lim_{x\to+\infty}\frac{(x^2+1)^5(\sqrt{x}-1)}{4}$$

$$= \frac{+\infty}{4} = +\infty$$

$$\lim_{x\to+\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}} = \lim_{x\to+\infty}\frac{\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{x}}}{\frac{\sqrt{2x+1}}{\sqrt{x}}}$$

$$= \lim_{x\to+\infty}\frac{1+\frac{\sqrt[3]{x}}{\sqrt{x}}+\frac{\sqrt[4]{x}}{\sqrt{x}}}{\sqrt{2+\frac{1}{x}}} = \lim_{x\to+\infty}\frac{1+\frac{\sqrt[6]{x^2}}{\sqrt[6]{x^3}}+\frac{\sqrt[4]{x}}{\sqrt[4]{x^2}}}{\sqrt{2+\frac{1}{x}}}$$

$$= \lim_{x\to+\infty}\frac{1+\sqrt[6]{\frac{1}{x}}+\sqrt[4]{\frac{1}{x}}}{\sqrt{2+\frac{1}{x}}} = \frac{1}{\sqrt{2}}$$

$$\lim_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$$

$$= \lim_{x\to+\infty}\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$$; $\div\sqrt{x}$ tenemos:

$$= \lim_{x\to+\infty}\frac{\sqrt{1+\sqrt{\frac{1}{x}}}}{\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}+1} = \frac{1}{2}$$